

# The construction of the first origami level-4 Menger sponge

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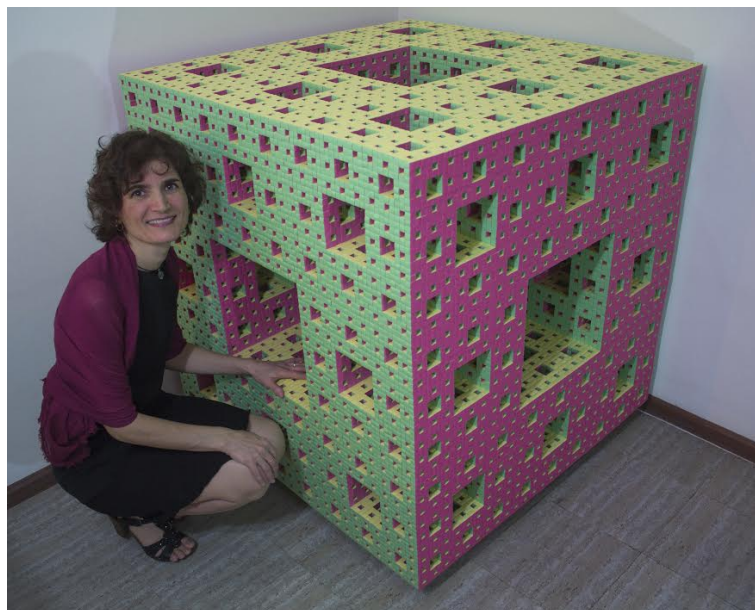
keywords: Origami, Menger sponge, Paper strips

## Abstract

The Menger sponge, first described by Karl Menger in 1926, is a 3-dimensional fractal with zero volume and infinite area. The first level of the Menger sponge is obtained by dividing a cube into 27 smaller cubes after which the cube in the middle and those at the center of each face are removed (20 cubes remain). To obtain the second level we repeat the process with each new cube, and so on. Repeating this process infinitely often gives the Menger sponge.

The first level-3 Menger sponge made in origami was completed in 2005 by Jeannine Mosely ([Mosely, 2015](#)) with 200 volunteers. Her method consisted in making cubes by folding business cards and then joining them together. Several level-3 sponges have been constructed since then, using the technique introduced by Mosely, but nobody ever managed to construct a level-4 origami Menger sponge.

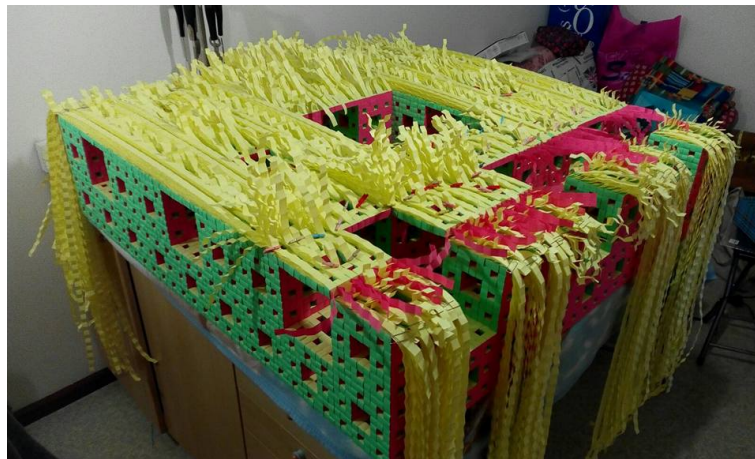
In this paper I will present a new technique that allowed me to build the first specimen of a level-4 origami Menger sponge. The result is shown in Figure 1.



**Figure 1:** Level-4 Menger sponge.

My technique can be used to construct models in origami composed of small cubes, which I call the elementary cubes. Every elementary cube is constructed by weaving together three strips in the three spatial directions. Each strip, pleated in squares, has to cover a specific number of adjacent cubes, depending on where in the model the strip is located. If the strip covers  $n$  cubes then it has to be pleated in  $2n + 4$  squares.

For the level-4 sponge, I used around 21 km of paper strips, 1.2 cm wide, in three different colors. Using strips of this size has allowed me to construct Menger sponges that are much smaller and lighter than the ones constructed with bussiness cards. Indeed, the edges of my level-4 sponge are slightly more than 1 m long and it weighs around 25 kg. It took me 15 months of work. It was completed on November 13th, 2016. Figure 2 shows an intermediate stage of the work.



**Figure 2:** Work in progress.

In the figure we see a lot of strips sticking out of the model. They are later folded with other strips, to form new elementary cubes.

For the level-4 Menger sponge I used strips of nine different lengths, from 6 to 166 squares.

Using the same technique I realized several level-2 and level-3 Menger sponges and I created puzzles and dynamic origami models. Currently I am working on a small level-3 Menger sponge using strips of width 5 mm. The resulting cube will have edges of length a bit less than 15 cm. Note that using smaller strips makes the construction much more difficult.

## References

Jeannine Mosely. Crowdsourcing origami sculptures. In *Origami<sup>6</sup> : II. Technology, Art, Education*, pages 625–634. American Mathematical Society, 2015.